

Sensor Aided Inertial Navigation Systems

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Outline

Today, I will discuss:

- **Inertial Navigation System.**
- Carrier phase differential GPS (CDGPS) - INS.
- CDGPS - Vision - INS.
- Stationary updates - INS.
- Near Real Time estimation.



Inertial Sensors

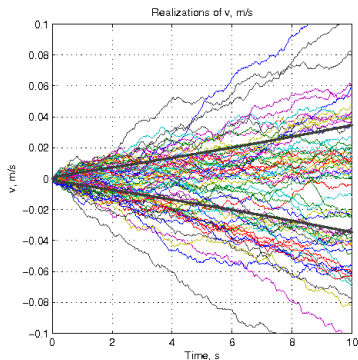
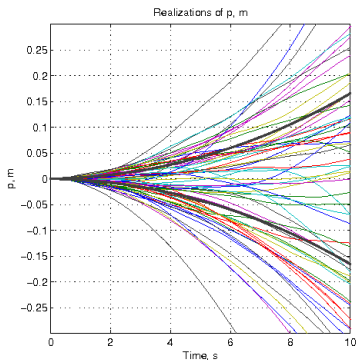
Popularity of inertial Sensors are due to:

- Immunity to jamming, No external reference required (in theory).
- Advent of MEMS sensors [1]:
 - Low cost, Small footprint ($\sim 22 \times 33 \times 11$ mm).
 - Well understood and quantifiable error models [5], [8], [12].
 - High frequency updates (> 100 Hz), High Bandwidth (> 330 Hz), High operating ranges ($\sim \pm 400$ deg/s, $\pm 10g$ m/s/s).
- Supplies full 6 degrees-of-freedom pose information.
- Consumer driven demand for applications such as
 - Navigation: routing, vehicle guidance & control [4], [19] etc.
 - High accuracy mobile mapping [18], [17].
 - Life-critical systems: Vehicle collision avoidances, automotive air bags etc.
 - Hand-held devices: Cellphones, Cameras, Electronics readers etc.



Inertial Sensors

- Potentially unbounded error growth in dead reckoning Inertial systems.





Aiding Sensors

“GPS, Vision, LIDAR, Magnetometer, Stationary updates etc.”.

- Features usually complement inertial sensors:
 - Independent and bounded long term errors.
 - **Low update frequency.**
 - **Do not provide 6-DOF information.**
- Aided INS problems can be formulated and solved under the Bayesian framework:
 - Extended Kalman Filters, Particle Filters, Unscented Kalman Filters etc.,

Inertial Navigation System

- Navigation state:

$$\mathbf{x}^\top = \begin{bmatrix} {}^n\mathbf{p}_{nb}^\top & {}^n\mathbf{v}_{nb}^\top & {}^n\mathbf{b}\mathbf{q}^\top \end{bmatrix}, \quad \mathbf{x} \in \mathbb{R}^6 \times \mathbb{S}^3.$$

- Inertial Measurements:

$${}^b\tilde{\mathbf{u}} = {}^b\mathbf{u} + {}^b\mathbf{b} + \mathbf{n}, \quad {}^b\mathbf{u} \in \mathbb{R}^6.$$

- Bias Gauss-Markov model:

$${}^b\dot{\mathbf{b}} = -\Lambda {}^b\mathbf{b} + \mathbf{n}_b.$$

- Kinematic equations:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{u})$$

$${}^n\dot{\mathbf{p}}_{nb} = {}^n\mathbf{v}_{nb}$$

$${}^n\dot{\mathbf{v}}_{nb} = {}^n_b\mathbf{R} {}^b\mathbf{f}_{ib} - {}^n\mathbf{g} - 2[{}^n\boldsymbol{\omega}_{ie} \times] {}^n\mathbf{v}_{nb}$$

$${}^n_b\dot{\mathbf{R}} = {}^n_b\mathbf{R} \left([{}^b\boldsymbol{\omega}_{ib} \times] - [{}^b\boldsymbol{\omega}_{ie} \times] \right)$$

- INS augmented error state: $\delta\bar{\mathbf{x}}^\top = \begin{bmatrix} \delta\mathbf{x}^\top & {}^b\delta\mathbf{b}_g^\top & {}^b\delta\mathbf{b}_a^\top \end{bmatrix} \in \mathbb{R}^{15}.$

- Linearized error propagation model: $\delta\dot{\bar{\mathbf{x}}} = \mathbf{A}\delta\bar{\mathbf{x}} + \mathbf{GN}.$

Frames: n=navigation, e=ECEF, i=inertial, b=body, c=camera

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Carrier Phase Differential GPS - INS

Tightly coupled GPS-INS is the *de-facto* standard for outdoor navigation:

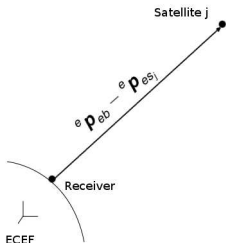
- Performance with double differenced, differential carrier-phase processing with differential corrections: [6, 7, 8],.
 - 1σ positioning accuracy in the order of 0.01 – 0.1 m.
 - 1σ attitude accuracy in the order of 1 deg.
- Well understood conditions to achieve full state observability [3, 8, 9, 10, 11, 16].
- Updates with a minimum of 2 satellite measurements (Loosely coupled needs at least 4).
- Can do sequential updates ($\mathbf{HPH}^T + \mathbf{R}$ is a scalar).



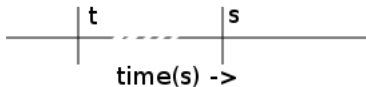
Carrier Phase Differential GPS - INS

Generic GPS measurement model:

$$\begin{aligned}\tilde{\rho}_k^j &= \|\mathbf{e}^e \mathbf{p}_{eb} - \mathbf{e}^e \mathbf{p}_{es_j}\| + \nu_\rho^j & ; & \nu_\rho^j \sim \mathcal{N}(0, 0.5^2) \\ \tilde{\phi}^j &= \|\mathbf{e}^e \mathbf{p}_{eb} - \mathbf{e}^e \mathbf{p}_{es_j}\| + \lambda N^j + \nu_\phi^j & ; & \nu_\phi^j \sim \mathcal{N}(0, 0.01^2) \\ \tilde{D}^j &= \frac{d}{dt} \|\mathbf{e}^e \mathbf{p}_{eb} - \mathbf{e}^e \mathbf{p}_{es_j}\| + \nu_D^j & ; & \nu_D^j \sim \mathcal{N}(0, 0.02^2)\end{aligned}$$



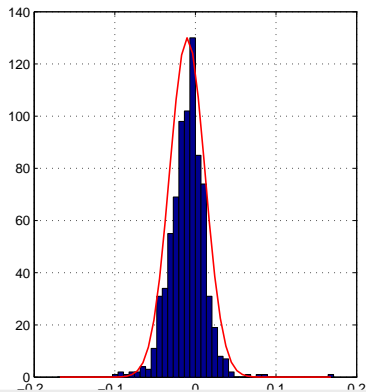
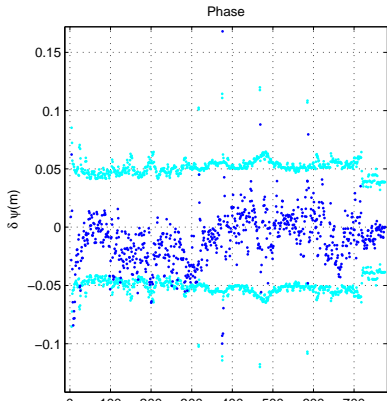
$$\begin{aligned}\delta\phi^j &= \tilde{\phi}^j - \hat{\phi}_k^j = \|\mathbf{e}^e \mathbf{p}_{eb} - \mathbf{e}^e \mathbf{p}_{es_j}\| - \|\mathbf{e}^e \hat{\mathbf{p}}_{eb} - \mathbf{e}^e \mathbf{p}_{es_j}\| + \nu_\phi^j \\ &= \mathbf{h}^j \delta \mathbf{x} + \nu_\phi^j\end{aligned}$$





Carrier Phase Differential GPS - INS

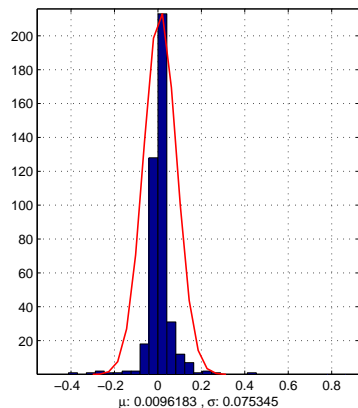
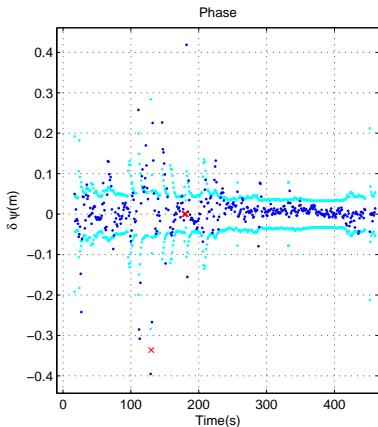
Example EKF Phase residuals for Satellite 3, Stationary rover, baseline: 7 km





Carrier Phase Differential GPS - INS

Example EKF Phase residuals for Satellite 18 driving on I-215:



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CDGPS - Vision - INS

Tightly coupled CDGPS - Vision - INS has several advantages:

- Under some well defined conditions, can contain drifts in:
 - Velocity and gyroscope bias.
 - Certain directions in attitude and accelerometer biases.
- Provides updates even with a single feature (unlike a loosely coupled system).
- Not computationally expensive like SLAM.
- Can be performed in real-time unlike Bundle adjustment.
- Can naturally extend to applications like Mapping, Surveying etc.

“Need to calibrate transformation from Body to Camera frames”.

$$\delta \mathbf{x}_*^T = \left[\begin{array}{cc|cc|cc} n_\delta \mathbf{p}_{nb}^T & n_\delta \mathbf{v}_{nb}^T & n_\rho^T & b_\delta \mathbf{b}_g^T & b_\delta \mathbf{b}_a^T & b_\delta \mathbf{p}_{bc}^T & b_\rho^T \end{array} \right] \in \mathbb{R}^{21}.$$

CDGPS - Vision - INS

- Feature vector in the Camera frame ${}^c \mathbf{p}_{cf_j}^\top = [x_j \quad y_j \quad z_j]$.
- Ideal perspective projection model:

$${}^c \mathbf{q}_{cf_j}^\top = [u_j \quad v_j] = \frac{1}{z_j} [x_j \quad y_j] \quad (5.1)$$

- Non-ideal Camera model [2]:

$${}^c \tilde{\mathbf{q}}_{cf_j}^\top = [f_x x'_j + c_x \quad f_y y'_j + c_y] + \mathbf{n}_c \quad (5.2)$$

where

$$x'_j = u_j(1 + k_1 r^2 + k_2 r^4) + 2p_1 u_j v_j + 2p_2(r^2 + 2u_j^2)$$

$$y'_j = v_j(1 + k_1 r^2 + k_2 r^4) + 2p_2 u_j v_j + 2p_1(r^2 + 2v_j^2)$$

$$r = \| {}^c \mathbf{q}_{cf_j} \|_2$$



Observability Analysis

Key results [14]:

Proposition 1

Assuming that the Camera is fully calibrated (i.e. $({}^b\mathbf{p}_{bc}, {}^b\mathbf{R}_c)$ are known), then the INS error state $\delta\bar{\mathbf{x}}(0)$ is fully observable with $N_0 \geq 3$ measurements at 3 time instants such that the set of points $\{{}^n\mathbf{p}_{nf_0}, \dots, {}^n\mathbf{p}_{nf_{N_0}}, {}^n\mathbf{p}_{nc_k}\}$ are not coplanar for all $0 \leq k \leq 2$.

Proposition 2

If the rover, initially at rest, is accelerates along a straight line and comes back to rest, aided by both GPS and Vision with $N_0 \geq 3$ features, then the observability gramian has full column rank. Therefore we have full state observability.



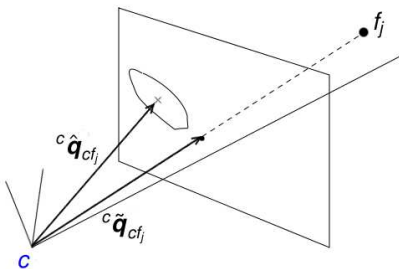
CDGPS-Vision-INS

Demo



CDGPS - Vision - INS

Data association:



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Stationary updates - INS

- Reset velocity (${}^b\mathbf{v}_{nb} = \mathbf{0}$) and rotation (${}^b\boldsymbol{\omega}_{nb} = \mathbf{0}$) to zero when system is at rest.

$$\delta\bar{\mathbf{x}}^T = \begin{bmatrix} {}^n\delta\boldsymbol{\rho}_{nb}^T & {}^n\delta\mathbf{v}_{nb}^T & {}^n\boldsymbol{\rho}_{nb}^T & {}^b\delta\mathbf{b}_g^T & {}^b\delta\mathbf{b}_a^T \end{bmatrix}$$

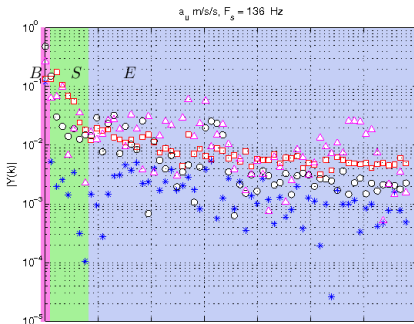
- Given stationarity, stationary updates or zero updates are preferable.
- Stationary updates corrects errors in:
 - velocity
 - gyroscope biases
 - some linear combination of attitude and accelerometer biases.
- Helps contain errors in position.
 - position
- *Detection of stationarity is a challenge.*
 - False detection introduces errors in sensor bias estimates directly.



Stationary updates - INS

Measurement model:

$${}^b\tilde{\mathbf{y}}_i = \mathbf{s}(iT_s) + \mathbf{e}(iT_s) + {}^b\mathbf{b}(iT_s) + \boldsymbol{\nu}(iT_s) + \mathbf{n}(iT_s)$$



Component	Region in the DFT (f Hz)
B	0
E	[8, 85]
S	(0, 10)

Legend:

Symbol	:	Maneuver
Blue asterisks	:	Stationary
Red squares	:	Decelerating
Black circles	:	Accelerating
Magenta triangles	:	Constant speed



Stationary updates - INS

- For each sensor k , choose appropriate $m(k) \in \mathbf{S}$ [13].
- ${}^k\varphi_{m(k)} \in \mathbb{C}$, ${}^k\varphi_{m(k)} \sim \mathcal{N}(\boldsymbol{\mu}, \mathbf{P})$.
- Define $f : \mathbb{R}^2 \rightarrow [0, +\infty)$ as

$$f({}^k\varphi_{m(k)}) = {}^k\varphi_{m(k)}^\top \mathbf{P}^{-1} {}^k\varphi_{m(k)}$$

- Under stationarity, $f({}^k\varphi_{m(k)})$ is an i.i.d. Rayleigh random process with $\chi = 1$.

Stationarity test

Given a chosen harmonic, $m(k) \in \mathbf{S}$, for each sensor k , the rover is stationary if

$$\max_{k \in \{1, \dots, 6\}} f({}^k\varphi_{m(k)}) < \lambda^2$$

for a chosen threshold $\lambda \in \mathbb{R}^+$.



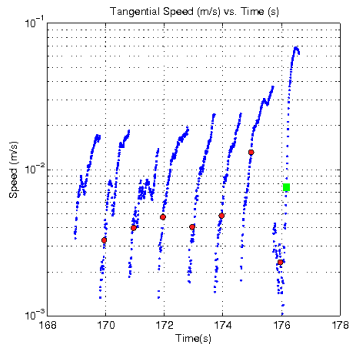
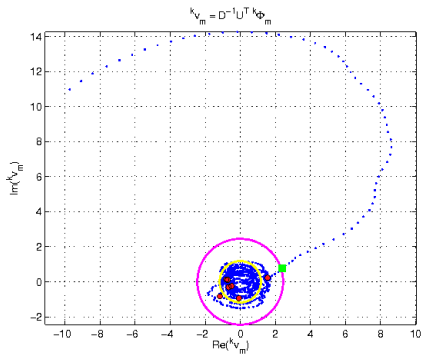


Stationary updates - INS

- Enables determination of λ using stochastic principles:
 - If $\lambda = 2.4477$, the $p\{\max_{k \in \{1, \dots, 6\}} f^{(k)} \varphi_{m(k)} < \lambda^2\} = 0.7351$.
 - Conservative choice of $\lambda = 0.05$ resulting in $p_{d/s} = 0.001$ (1 detection every 5 s when $F_s = 130$ Hz).
- Upper bounds on probability of false detection [13].



Stationary updates - INS





Stationary updates - INS

Observability analysis [15]:

$\delta \mathbf{x}$	S	M - S	S - M - S, ($\omega = 0$)	S - M - S
${}^n \delta \mathbf{p}_{nb}$	\mathbb{R}^3	\mathbb{R}^3	\mathbb{R}^3	\mathbb{R}^3
${}^n \delta \mathbf{v}_{nb}$	0	$\mathbf{M}(t_1) \mathbf{e}_i$	0	0
${}^n \delta \boldsymbol{\rho}_{nb}$	\mathbf{e}_i	\mathbf{e}_i	\mathbf{e}_i	${}^n \mathbf{g}$
${}^b \delta \mathbf{b}_g$	0	0	0	0
${}^b \delta \mathbf{b}_a$	${}^b_n \mathbf{R} [{}^n \mathbf{g} \times] \mathbf{e}_i$	${}^b_n \mathbf{R}(t_1) [{}^n \mathbf{g} \times] \mathbf{e}_i$	${}^b_n \mathbf{R} [{}^n \mathbf{g} \times] \mathbf{e}_i$	0

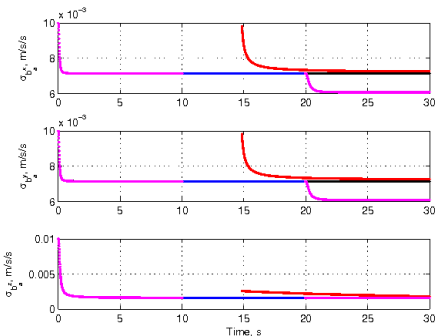
Fixed point optimal smoother:

$$\delta \hat{\mathbf{x}}_0 = E\{\delta \hat{\mathbf{x}}_0 | \delta \mathbf{y}_1 \dots \delta \mathbf{y}_M\}.$$



Stationary updates - INS

$\delta \mathbf{x}$	S	$M - S$	$S - M - S, (\omega = 0)$	$S - M - S$
${}^b \delta \mathbf{b}_a$	${}^b_n \mathbf{R} [{}^n \mathbf{g} \times] \mathbf{e}_i$	${}^b_n \mathbf{R}(t_1) [{}^n \mathbf{g} \times] \mathbf{e}_i$	${}^b_n \mathbf{R} [{}^n \mathbf{g} \times] \mathbf{e}_i$	$\mathbf{0}$



Legend:

Color	Maneuver
Blue	S
Red	$M - S$
Black	$S - M - S (\omega = 0)$
Magenta	$S - M - S (\omega \neq 0)$

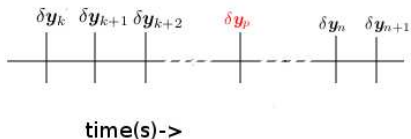


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Near Real Time estimation



- Unlikely residuals: $\delta \mathbf{y}_p^\top \mathbf{S}_p^{-1} \delta \mathbf{y}_p > \lambda$.
- Validation from measurements in future time $\tilde{\mathbf{y}}_n, \tilde{\mathbf{y}}_{n+1}, \dots$
- Updating $\delta \bar{\mathbf{x}}_n$ with $\delta \mathbf{y}_p$ violates “white-noise” assumption.

A possible solution:

- Append state vector: $\delta \alpha^\top = [\delta \bar{\mathbf{x}}_p^\top \quad \delta \bar{\mathbf{x}}_n^\top]$.
- Append the covariance: $\mathbf{P}_\alpha = \begin{bmatrix} \mathbf{P}_p & E\{\delta \bar{\mathbf{x}}_p \bar{\mathbf{x}}_n^\top\} \\ E\{\bar{\mathbf{x}}_n \delta \bar{\mathbf{x}}_p^\top\} & \mathbf{P}_n \end{bmatrix}$.
- Update: $\delta \mathbf{y}_p = [\mathbf{h}_p \quad \mathbf{0}] \delta \alpha$.



Thanks for listening!



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