



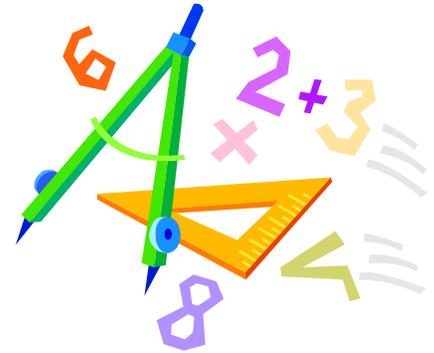
Navigating by the Numbers



Unit Topic:	Navigation
Grade Level:	7 th grade (with suggestions to scale for grades 6 to 8)
Lesson No.	3 of 10
Lesson Subject(s):	Calculation, approximation, geometry, trigonometry, triangulation
Key Words:	Trigonometry, Geometry, Surveying

Lesson Abstract —

In this lesson, students will learn that math is important in navigation and engineering. Ancient land and sea navigators started with the most basic navigation equations ($\text{Speed} \times \text{Time} = \text{Distance}$). Today, navigational satellites use equations that take into account the relative effects of space and time. However, even these high tech wonders cannot be built without pure and simple math concepts — basic geometry and trigonometry — used for thousands of years. These basic concepts will be discussed and illustrated using the following activities:



- *Stay In Shape* – students will learn about how distances on the surface of the earth are actually arcs, and not just straight lines. They will also learn how to calculate the arc length.
- *Trig River* – students will learn how to do basic surveying. Also, they will learn how to determine the width of a river without crossing it.

Lesson Opening Topics / Motivation —

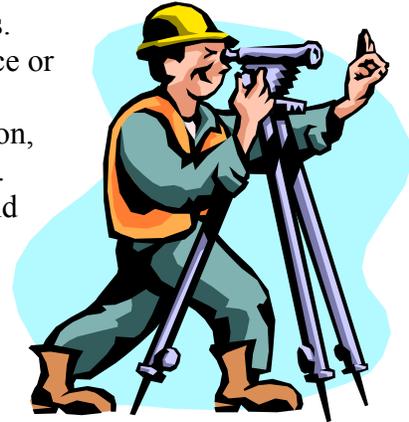
How important is math in navigation? (Possible answer: It depends on the goal of the traveler.) If you have unlimited time and your destination is visible from miles away, you may not need math. But if you want to get somewhere as fast as possible or the destination is not visible until you are on top of it, math is essential.

You can navigate your home without doing any math because there are visual clues to guide you. You can even drive from one city to another without math because signs have been put up to direct drivers. On the other hand, if you are trying to cross an ocean where there are not any landmarks, you could not navigate accurately without math. For the Polynesians crossing the Pacific Ocean, approximate methods worked because they could use visual clues (i.e., birds, waves, stars) once they were within 100 miles or so of their destination. Even then though they had to imagine angles, count multiples of the distances between stars, and visualize vectors of

wind and water currents. Essentially, these early Polynesian sailors were doing geometry in their heads.

As the world grows more complex, we want to navigate faster and know exact distances and times of travel. How did people first determine distances between cities? Or, how about the size of the Earth? (Answer: They measured the distance between two cities and then used trigonometry to extrapolate that to the size of the earth.) Is it possible to determine the width of a river without crossing it? (Answer: Yes, but it requires that you understand trigonometry.)

It is all possible using relationships of simple shapes and angles. These principles can be applied to determine almost any distance or height given appropriate reference points. At the heart of both navigation and engineering is the art and science of measuring on, near, or beneath the surface of the earth — known as surveying. Surveyors are able to determine very exact relative distances and heights, and this allows precise positioning of anything being built, greatly improving strength, accuracy, and even the safety of the final product. A survey technician is needed in many jobs, including: the forest service, city and road planning, building construction, cartography (map making), mining, and even building satellites!



Lesson Desired Student Outcomes —

Students will understand that navigation and engineering are based on mathematics. The math can be a simple mental estimation or a complex equation used for a precise calculation.

Science: Students should be able to:

- Predict (hypothesize). (1)
- Use protractors to measure angles. (1)
- Use meter sticks to measure metric distances. (1)
- Show how surveyors use math and science. (5)

Math: Students should be able to:

- Use numbers (including pi) to count, measure, label, and indicate distances on a map. (1)
- Calculate the circumference of a circle. (5)
- Solve problems using the circumference of a circle. (4)
- Calculate rates of speed. (5)
- Use the Pythagorean Theorem to solve real-world problems. (2,4)
- Recognize right, isosceles, and equilateral triangles. (4)
- Use right triangle trigonometry. (4)
- Create a bar graph (extension activity). (3)
- Convert from US Customary to Metric units (extension activity). (6)
- Estimate distances. (5)

- Work with ratios. (6)

Colorado State Standards Met

<http://www.mcrel.org/compendium/search.asp>

- Science Standard 1, 5
- Math Standard 1, 2, 3, 4, 5, 6

Lesson Background & Concepts for Teachers —

Triangles

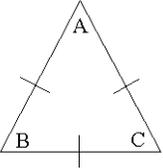
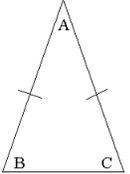
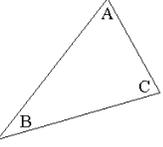
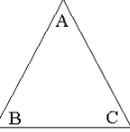
The sum of the angles in any triangle is 180° .

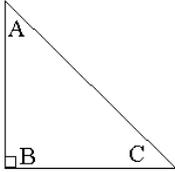
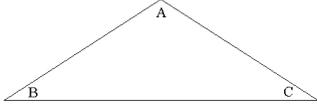
$$\angle A + \angle B + \angle C = 180^\circ$$

A great website for showing that the angles of a triangle add up to 180 degrees:

<http://www.utc.edu/~cpmawata/geom/geom2.htm>

Types of Triangles:

EQUILATERAL		All the sides are the same length (shown by the line through each of them) and all the angles are the same length. $\angle A = \angle B = \angle C = 60^\circ$
ISOSCELES		Two sides are equal (shown by the lines). Angles opposite the equal sides are equal. $\angle B = \angle C$
SCALENE		No sides are equal. No angles are equal. $\angle A \neq \angle B \neq \angle C$
ACUTE-ANGLED		All the angles are less than 90° . $\angle A < 90^\circ; \angle B < 90^\circ; \angle C < 90^\circ$

RIGHT- ANGLED		<p>One of the angles is 90°. This is shown by the square block at angle B. $\angle B = 90^\circ$</p>
OBTUSE- ANGLED		<p>One of the angles is greater than 90°. The longest side is opposite the largest angle. $\angle A > 90^\circ$</p>

Pythagoras' Theorem

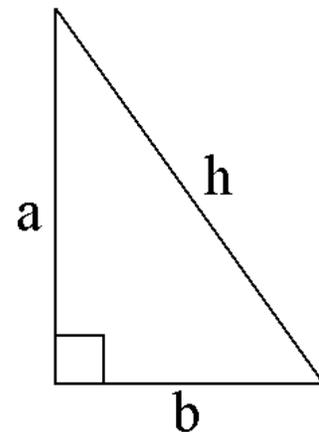
Pythagoras was a Greek mathematician who lived from 569 -500 B.C. It is said that he discovered this special property of right-angled triangles while looking at the tiles of an Egyptian Palace.

Pythagoras said, "In a right angled triangle, the area of the square on the hypotenuse equals the sum of the squares on the other two sides."

h is the hypotenuse

$$h^2 = a^2 + b^2$$

$$h = \sqrt{a^2 + b^2}$$



Pythagorean Theorem Visual Demonstrations:

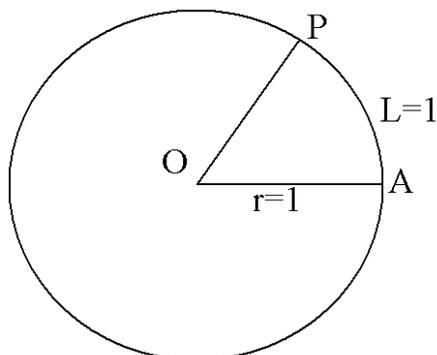
<http://www.utc.edu/~cpmawata/geom/geom6.htm>

<http://www.utc.edu/~cpmawata/geom/geom7.htm>

Circles

A radian is the angle made when the radius of a circle represents an arc on its perimeter.

One radian is the measure of the angle created at the center of a circle by an arc on the perimeter equal in length to the radius of the circle. A radian is a different way to measure an angle than using degrees.



If $r = 1$ unit
and length, $L = 1$ unit
then $\angle AOP = 1$ radian

$$1 \text{ radian} = \frac{180}{\pi} = \frac{180}{3.141...} = 57.30^\circ$$

1 radian = 57.30 degrees

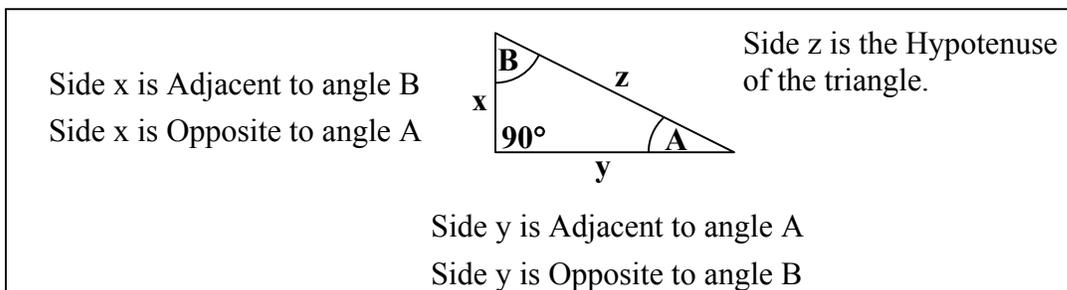
If there are 360° in a circle: $360^\circ / 57.3^\circ$ per radian = 6.28 radians on the perimeter of a circle. Notice that is just 2×3.14 (π or π) radians so 3.14 radians = 180° .

More descriptions and sample problems can be found at this website:

<http://www.staff.vu.edu.au/mcaonline/units/trig/trigraddegrees.html>

Trigonometry

Trigonometry is a branch of mathematics dealing with relationships of the angles and sides of triangles. The three basic trigonometric relations that we are concerned with — sine, cosine and tangent — are ratios of the lengths of two sides of a particular triangle. A very useful type of triangle is a *right* triangle, which has one angle equal to 90 degrees. By definition, the 90° angle is made by two lines that are perpendicular to each other (like the corner of a square), and the third side of the triangle is made by a sloping line connecting the two perpendiculars. This sloping line is called the *hypotenuse*, and the name comes from the Greek words *hypo* (meaning *under*) and *teinein* (meaning *to stretch*). Essentially, hypotenuse means *to stretch under the 90° angle*. It is easiest to show this visually.



<u>For Angle A:</u>		
$\sin(A) =$	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$= \frac{x}{z}$
$\cos(A) =$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$	$= \frac{y}{z}$
$\tan(A) =$	$\frac{\text{Opposite}}{\text{Adjacent}}$	$= \frac{x}{y}$

<u>For Angle B:</u>		
$\sin(B) =$	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$= \frac{y}{z}$
$\cos(B) =$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$	$= \frac{x}{z}$
$\tan(B) =$	$\frac{\text{Opposite}}{\text{Adjacent}}$	$= \frac{y}{x}$

The letters *SOH CAH TOA* can effectively help students remember which sides go with which functions (Sine = Opposite / Hypotenuse, Cosine = Adjacent/Hypotenuse, etc.). Mnemonics

may help students memorize the relations: “Some Old Hag Caught A Hippie Tripping On Art” or “Some Oaf Happily Cut A Hole Through Our Apartment.”

Lesson Vocabulary List —

- **Geometry** – The mathematical relationships of points, lines, angles, surfaces, and solids.
- **Trigonometry** – The mathematical relationships between the sides and the angles of triangles.

Activity Attachments —

[Activity 1: Stay in Shape](#) – Students learn how understanding geometric shapes like triangles and circles is fundamental to understanding navigation.

[Activity 2: Trig River](#) – Students use trigonometry to determine the width of a “river” without crossing it!

Lesson Closure and Follow-up —

Without math at some level, most navigation is random wandering and luck. Even using the sun as a guide point means you are using geometry: you must have a mental picture of a plane (the earth), a reference point (the sun), and the angle away from the reference point toward your destination. Understanding the mathematical relationships of shapes, angles, and the physical universe allows navigation to become efficient and safe rather than just hoping to reach a destination. Even the simplest equation, Speed x Time = Distance, can be a mental guess (say roughly 20mph x about $\frac{3}{4}$ hour = 15 miles) or a precise calculation (21.3257mph x 44.3552 hours = 15.7651 miles). Whether the goal is finding a continent across an ocean or determining a satellite orbit to within centimeters, math is the vehicle that takes you there.

Lesson Extension Activities —

- Use the latitude and longitude of two cities on the globe and find how far apart they are: <http://jan.ucc.nau.edu/~cvm/latlongdist.php>
- Estimate the size of the Earth: Use the link above to find the distance between two cities that are on the same longitude line. By knowing the difference in latitude between the cities and that the circumference of the Earth has 360° of latitude, the students should be able to come up with an estimate of the Earth’s circumference! (Answer: Divide the distance between cities by the difference in latitude (degrees) so that the distance per degrees of latitude is known. 360° multiplied by this distance/° should give a number very close to the circumference of the Earth at the equator. (Answer: About 24,900 miles or 40,070 km.) (Derived from http://www.math.wisc.edu/~rushton/scopes_kti/northstar/main.html)
- Have students research other methods of determining the circumference or radius of the Earth. Eratosthenes (276-194 BC) was a Greek scholar who was the first person to determine the circumference of the Earth. Many additional methods have been found since the hard work of Eratosthenes.

Lesson Assessment and Evaluation —

Pre-Lesson Assessment

- Discussion Question:
 - How important is math in navigation? (Answer: It depends on the goal of the traveler. If you have unlimited time and your destination is visible from miles away, you may not need math. But, if you want to get somewhere as fast as possible or the destination is not visible until you are practically on top of it, then math is essential.)

Post-Introduction Assessment

- Voting (record number of votes on board or piece of paper):
 - Thumbs Up: if you think it is good to use math when navigating.
 - Thumbs Down: if you think navigation can be close enough without math.

Post-Lesson Assessment

- Voting (compare votes to Post-Introduction assessment; hopefully the thumbs up increased):
 - Thumbs Up: if you think it is good to use math when navigating.
 - Thumbs Down: if you think navigation can be close enough without math.
- Bingo: Provide each student with a sheet of paper containing a list of the lesson vocabulary terms. Have each student walk around the room and find a student who can define one vocabulary term. Students must find a different student for each term. When a student has all terms completed s/he shouts “Bingo!” Continue until all students bingo.

Homework

Internet search: Have students research lesson concepts on the Internet and lead a brief class discussion of student findings during next class period.

Lesson References —

Check out these other interesting websites for more information:

<http://www.utc.edu/~cpmawata/geom/geom2.htm>

<http://www.utc.edu/~cpmawata/geom/geom6.htm>

<http://www.utc.edu/~cpmawata/geom/geom7.htm>

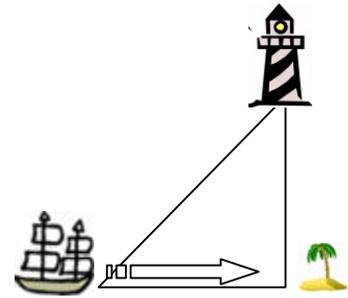
<http://www.staff.vu.edu.au/mcaonline/units/trig/trigraddegrees.html>

Activity: Stay in Shape

This activity is planned for 28 students working individually.

Activity Materials List —

- [Shape Worksheet](#)
- [Shape Datasheet](#)



Activity Equipment and Tools List —

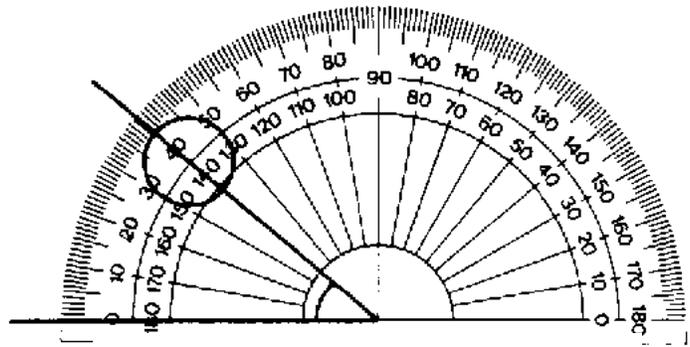
- Pencils
- Protractors

Activity Cost Estimate —

\$0 (Note: If you do not have protractors, make copies of the paper half protractor and have the students make their own. [Paper Half protractors.](#))

Activity Attachments —

- [Shape Worksheet](#)
- [Shape Datasheet](#)



Protractors are used to measure angles.

Activity Time Estimate —

40-50 min.

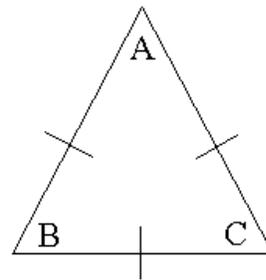
Activity Procedure —

A. Background:

Geometry: Triangles

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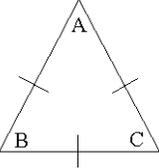
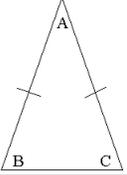
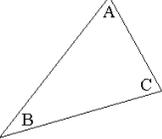
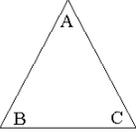
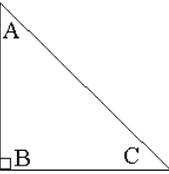
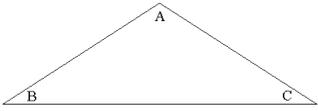
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Types of Triangles

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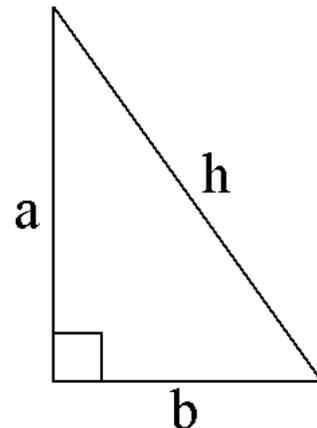
Pythagoras' Theorem

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h is the hypotenuse

$$h^2 = a^2 + b^2$$

$$h = \sqrt{a^2 + b^2}$$



Pythagorean Theorem Visual Demonstrations

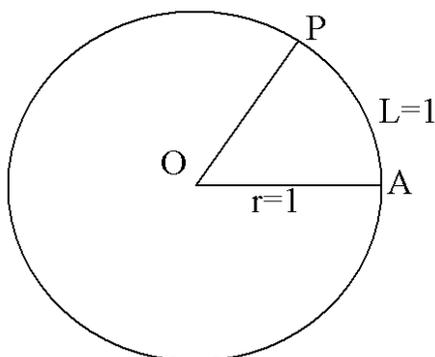
<http://www.utc.edu/~cpmawata/geom/geom6.htm>

<http://www.utc.edu/~cpmawata/geom/geom7.htm>

Circles

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One radian is the measure of the angle created at the center of a circle by an arc on the perimeter equal in length to the radius of the circle. A radian is a different way to measure an angle than using degrees.



If $r = 1$ unit
and length, $L = 1$ unit
then $\angle AOP = 1$ radian

$$1 \text{ radian} = \frac{180}{\pi} = \frac{180}{3.141\dots} = 57.30^\circ$$

1 radian = 57.30 degrees

If there are 360 degrees in a circle, then $360 \text{ degrees} / 57.3 \text{ degrees per radian} = 6.28$ radians on the perimeter of a circle. Notice that is just 2×3.14 (pi or π) radians so 3.14 radians = 180 degrees.

More descriptions and sample problems can be found at this website:

<http://www.staff.vu.edu.au/mcaonline/units/trig/trigraddegrees.html>

B. Before the activity:

1. Print out the worksheets.
2. Review the properties of triangles and circles.

C. With the Students:

Tell the students that understanding shapes and angles is important in navigation; being able to determine a position relative to other objects depends on knowing geometric relationships. Explain to the students that the word “geometry” comes from the Latin roots *geo* (meaning land or Earth) and *metry* (meaning measure). Geometry, therefore, means *measurements of the Earth*, which is precisely what is needed in navigation.

1. Have the students brainstorm in groups what they know about triangles and circles. Ask for answers, then write and clarify the true statements on the board.
2. Hand out the [Shape Worksheet](#) and [Shape Datasheet](#).

3. Do the first ship problem as a class. Have the students try the 2nd problem on their own.

Note: It may be helpful to write the following chart on the board for Ship #2. Write the numbers 1-15 and then have the students come up with the square values for each.

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Square	1	4	9	16	25	36	49	64	81	100	121	144	169	196	225

4. Do the Ship #3 problem with the class. (For problem number 3, the relationship between a circle's radius and its perimeter must be established. Discuss what this relationship is from the background section.)
5. Ask the students to add any new properties of triangles and circles learned to the list already on the board.

Math Skills Reinforced —

- 6th, 7th, 8th: This activity has students using geometric relations of triangles and circles to find distances. Students will use the circumference of a circle, calculate rates of speed, identify between right, isosceles, and equilateral triangles, and use the Pythagorean theorem.

Activity Troubleshooting Tips —

Some students might get confused when working on the circular problem (#3). Be sure to go through this problem with them. Spend time explaining the difference between radians and degrees. What does it mean to be 50 radians away? The worksheet should definitely be used as a teaching tool.

Activity Desired Student Outcomes —

After this activity, students will understand how the properties of triangles and circles can be used to find unknown distances.

Activity Assessment & Evaluation —

Pre-Activity Assessment

- Brainstorming: Brainstorm properties of triangles and circles. Write on board as directed in C1.

Activity Embedded Assessment

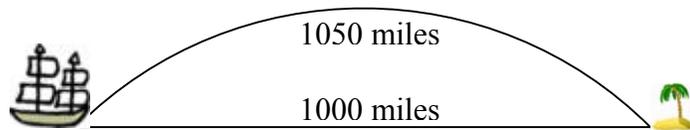
- Data Sheet and Work Sheet: Have students do geometry calculations on the data sheet of angle and distances.

Post-Activity Assessment

- Debrief: Ask for input from students to add any new properties of triangles and circles learned to the list already on the board.

Suggestions to Scale Activity for Grades 6 to 8 —

- 6th Grade: Work through all the activities together.
- 7th Grade: Do as is.
- 8th Grade: Discuss why the curve of the Earth was not used in the Ship #1 and #2 problems. (Answer: Because the distances were very small compared to the curve of the Earth. At small distances the Earth can be considered flat.) At what distance is it necessary to include the curve of the Earth? (Answer: It depends on the application and how accurate the measurement needs to be. But, if you were sailing to an island using a flat Earth map, you would be short of your target by about 5 miles after sailing 100 miles (probably within sight on a clear day). If the island were 1000 flat miles away, you would end up being 50 miles short! Time for an SOS!

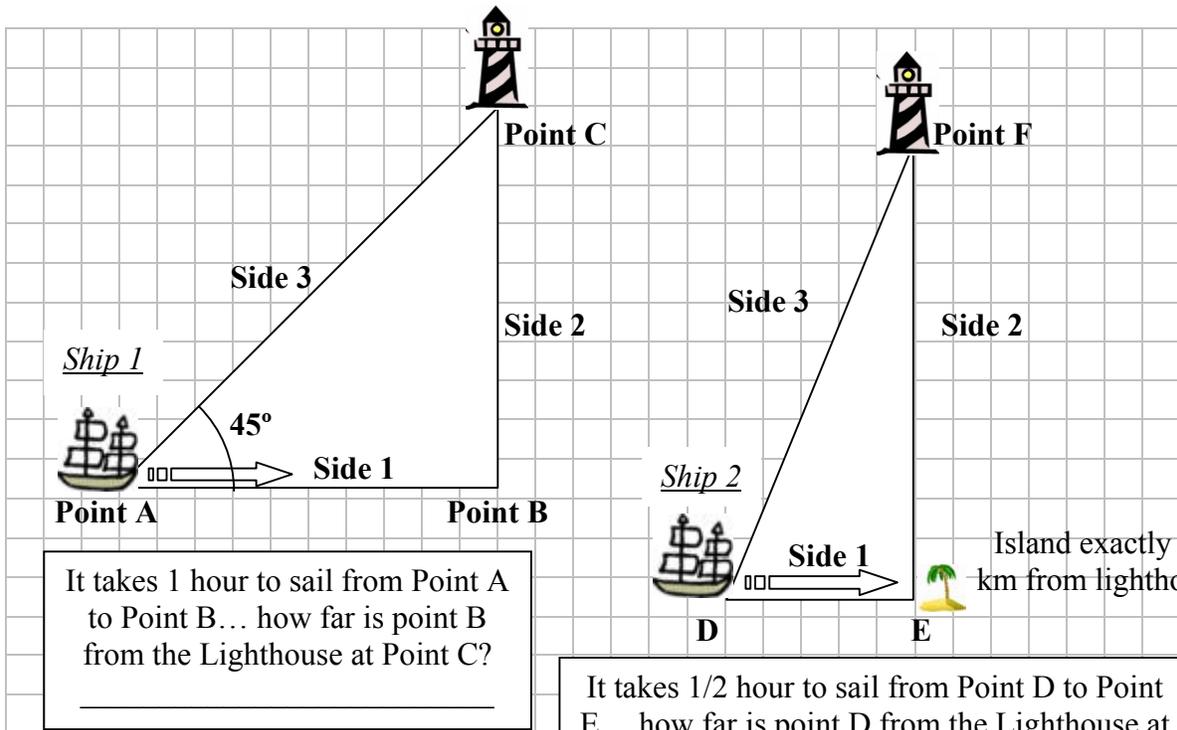


Keep in Shape!

Name: _____

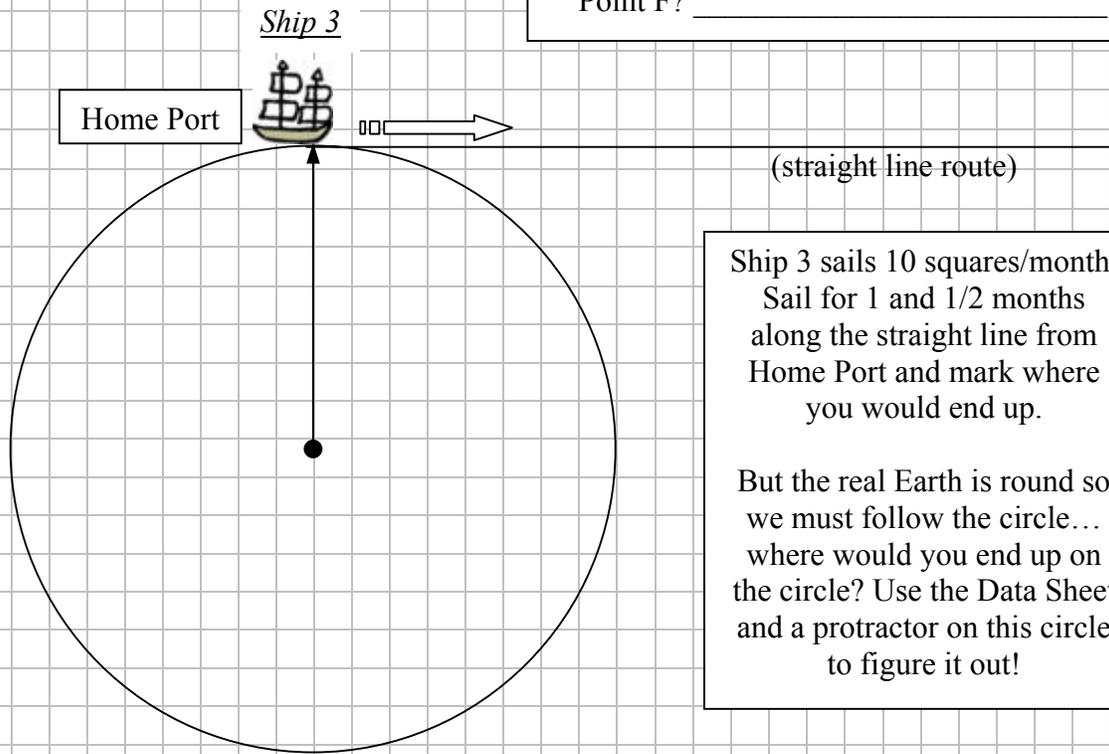
Date: _____

Ships 1 and 2 sail at a speed of 10 km (squares)/hour.



It takes 1 hour to sail from Point A to Point B... how far is point B from the Lighthouse at Point C?

It takes 1/2 hour to sail from Point D to Point E... how far is point D from the Lighthouse at Point F? _____



Ship 3 sails 10 squares/month. Sail for 1 and 1/2 months along the straight line from Home Port and mark where you would end up.

But the real Earth is round so we must follow the circle... where would you end up on the circle? Use the Data Sheet and a protractor on this circle to figure it out!

Keep in Shape Data!

Name: _____

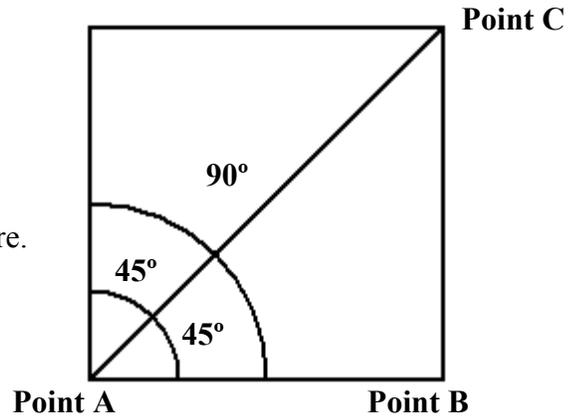
Date: _____

Ship 1:

First find the distance from Point A to Point B:

10 km/hour x 1 hour = _____ km

Next observe that the triangle we've made is $\frac{1}{2}$ of a square. Once all sides of a square have equal lengths, what is the length from Point B to Point C? _____ km



Ship 2:

First find the distance from Point D to Point E:

10 km/hour x 1/2 hour = _____ km

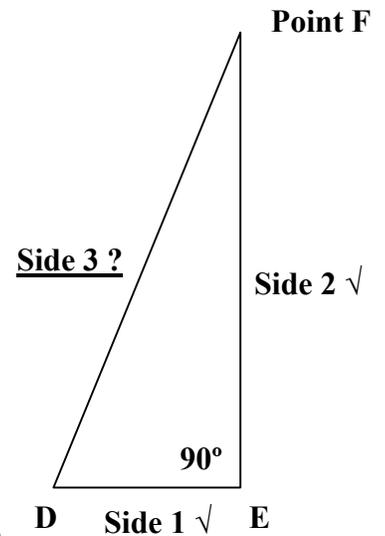
The buoy is 12 km from the lighthouse so we have the lengths of two sides of the triangle and we know one angle is 90° (this is a "right" triangle) – that means we can use the Pythagorean Theorem.

Side 1 x Side 1 + Side 2 x Side 2 = Side 3 x Side 3:

_____ x _____ + _____ x _____ = _____ x _____

Do the left side first:

_____ = _____ x _____ Side 3 = _____ (km)



Ship 3:

First find the radius of the circle in squares (count them on the worksheet). _____
This is the value of one radian for this circle.

So each square is what fraction of a radian? _____ (divide 1 by the number of squares)

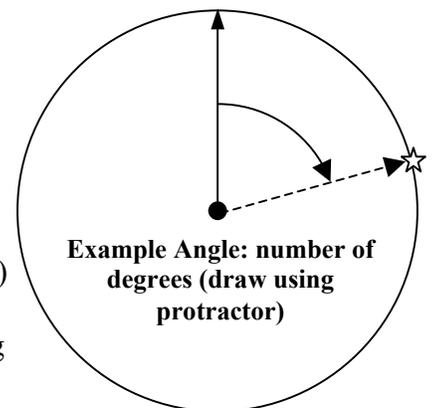
Convert the number of squares the ship traveled to radians:

Dist (squares) x radian fraction (radians/square) = _____ x _____ = _____ radians.

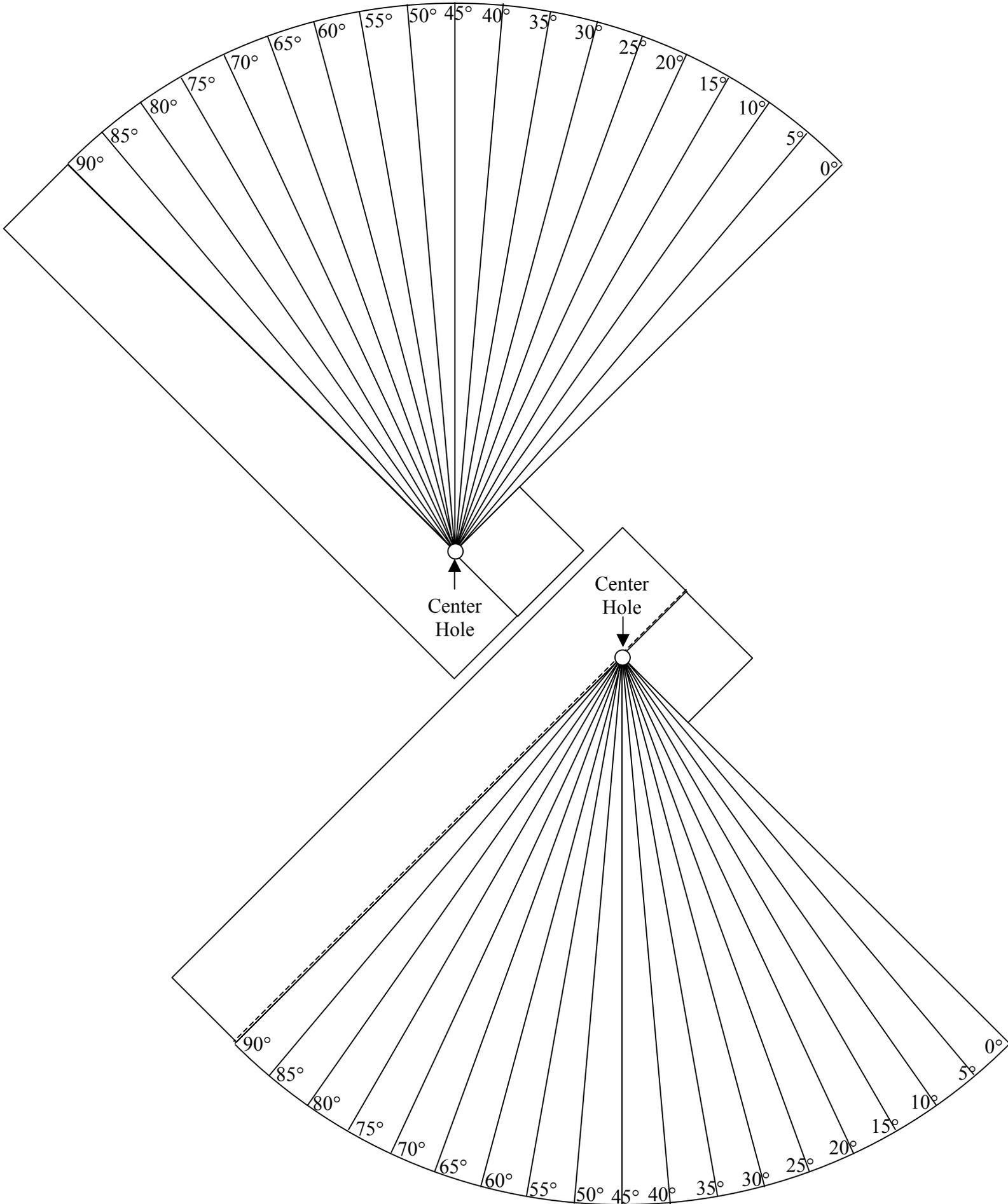
Since 3.14 (pi or π) radians = 180° (half a circle):

_____ (radians) x 180 (degrees) / 3.14 (radians) = _____ $^\circ$ (degrees)

Use a protractor to measure this number of degrees from the starting point radius of ship 3. That is where the ship ends up. (Note: the angle drawn in this circle is not correct...)



Half Protractors:



Activity: Trig River

This activity is planned for 28 students working in groups of two.



Activity Materials List —

- [Trig River Worksheet](#)
- Protractors
(or print out [paper half-protractors](#))
- 14 pieces of string (6-inch each) – any long very thin items could be substituted.

Activity Equipment and Tools List —

- Pencils
- Tape measure or meter stick.

Activity Cost Estimate —

\$0

Activity Attachments —

[Paper Half-Protractors](#)
[Trig River Worksheet](#)

Activity Time Estimate —

40-50 min.

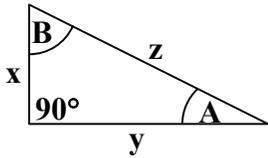
Activity Procedure —

A. Background:

Is it possible to determine the width of a river without crossing it? It is possible to come very close for a river of any size. The same principle used to determine the width of a river can be applied to other situations, including determining the height of a hill, a tree or a building and is used by surveyors to precisely position anything being built. Surveying is the art and science of measuring on, near, or beneath the surface of the earth. As part of an engineering team, a survey technician is responsible for measurements, calculations, and positioning involved in many areas including: city and road planning, building construction, cartography (map making), mining, and creating legal boundaries. The simple geometric shape that makes this all possible is the *triangle*.

Trigonometry is a branch of mathematics dealing with relationships between the angles and sides of triangles. The three basic trigonometric relations that we are concerned with in this activity are: sine, cosine, and tangent (abbreviated as sin, cos, and tan). They are the ratios of the lengths of two sides of a particular triangle. The particular type of

triangle that is most useful to mathematics is a *right* triangle, which has one angle equal to 90 degrees. By definition, the 90-degree angle is made by two lines that are perpendicular to each other (like the corner of a square), and a sloping line connecting the two perpendicular lines makes the third side of the triangle. This sloping line is called the *hypotenuse*, and the name comes from the Greek *hypo* (meaning under) and *teinein* (meaning to stretch). It is easiest to show this visually:

Side x is Adjacent to angle B Side x is Opposite to angle A		Side z is the Hypotenuse for both angles A and B
Side y is Adjacent to angle A Side y is Opposite to angle B		

<p><u>For Angle "A":</u></p> $\sin(A) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{x}{z}$ $\cos(A) = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{y}{z}$ $\tan(A) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{x}{y}$	<p><u>For Angle "B":</u></p> $\sin(B) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{y}{z}$ $\cos(B) = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{x}{z}$ $\tan(B) = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{y}{x}$
--	--

SOH CAH TOA. These letters can help students remember which sides go with which functions (Sine = Opposite / Hypotenuse, etc.). Mnemonics may help 6-8th graders memorize the relations: "Some Old Hag Caught A Hippie Tripping On Art" or "Some Oaf Happily Cut A Hole Through Our Apartment." SOH CAH TOA.

A navigation example using trigonometry is shown here:
<http://www.staff.vu.edu.au/mcaonline/units/trig/ratios.html>

B. Before the activity:

1. Print out the worksheets.
2. Decide whether activity will be performed indoors or outside.
3. Prepare or choose the objects that will be the markers and shore boundaries.

C. With the Students:

Is it possible to determine the width of a river without crossing it? (Allow discussion and entertain any creative ideas: walk through the river, throw a rope across, use stepping-stones, etc. Then discuss the state that this river that does not allow the solutions such as: it is too deep, the current is too swift, it is too wide, you do not have that tool, etc.) If a student knows about using triangles, have them explain as much as they can or introduce the idea and give a quick review of a right triangle — drawing and labeling it on the board.

Define the “river” for the students. For example, if working inside, rearrange desks to form the two “banks” of the river (with space for students to work on each “shore”). If working outside, choose a spot with two widely spaced (2-5 meters) and roughly parallel lines to define the “river” banks. For example, a wide sidewalk, two lines on a football field, or a strip of grass with straight edges. If a small “river” is being measured, have the students measure in centimeters, if a larger “river” is being used, have the students measure in meters. Because these are ratios of distances, the result at the end should have the same unit (meters, centimeters, etc.) used to make the initial measurement. The students will be making an estimate of how wide the river is.

1. On one side of the river (as close to the middle of the side as possible, set an object that will be the Far Edge Marker. Normally this represents a tree right at the edge of the opposite side of the river.
2. Directly across from the marker, place a Zero Edge Marker (see Figure 1). All the students should be on this side of the river.
3. Lay the measuring tape along the “zero edge bank” with one end at the Zero Edge Marker and place a piece of tape every $\frac{1}{2}$ meter on the river edge of the desks. Repeat this in the other direction (see Figure 1).

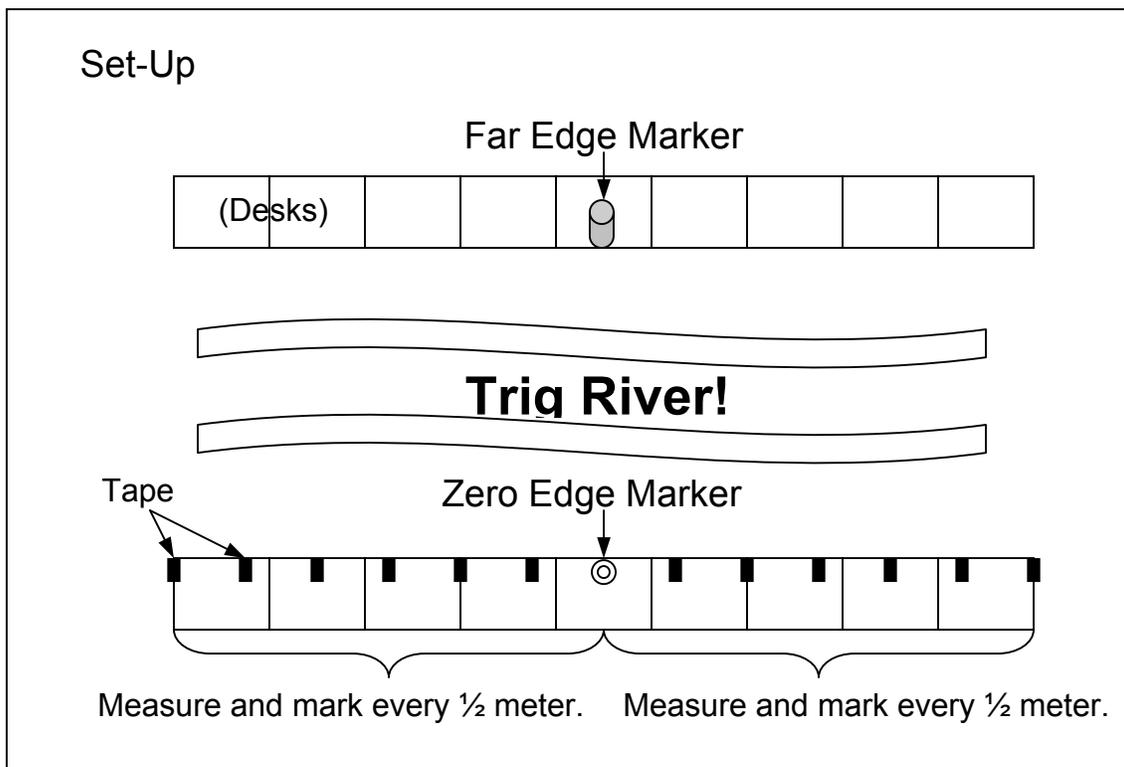


Figure 1 – Boundary and Markers

4. Give each student a [Trig River Worksheet](#).
5. Each group will work from a tape mark. When both students are at their mark and have written down the distance from the Zero Edge Marker on their worksheet, give them a protractor.
6. Lay the protractor with the center point on the tape and the zero angle pointing toward the Zero Edge Marker (see Figure 2).
7. One student will hold the protractor in place while the other places one end of the string on the center point of the protractor and aims the other end at the Far Edge Marker. Read the angle the string passes over on the protractor (counting up from zero; this should not be more than 90 degrees), and record it on the worksheet. While the students do this, the teacher can measure the actual distance between the two markers (but do not tell yet).

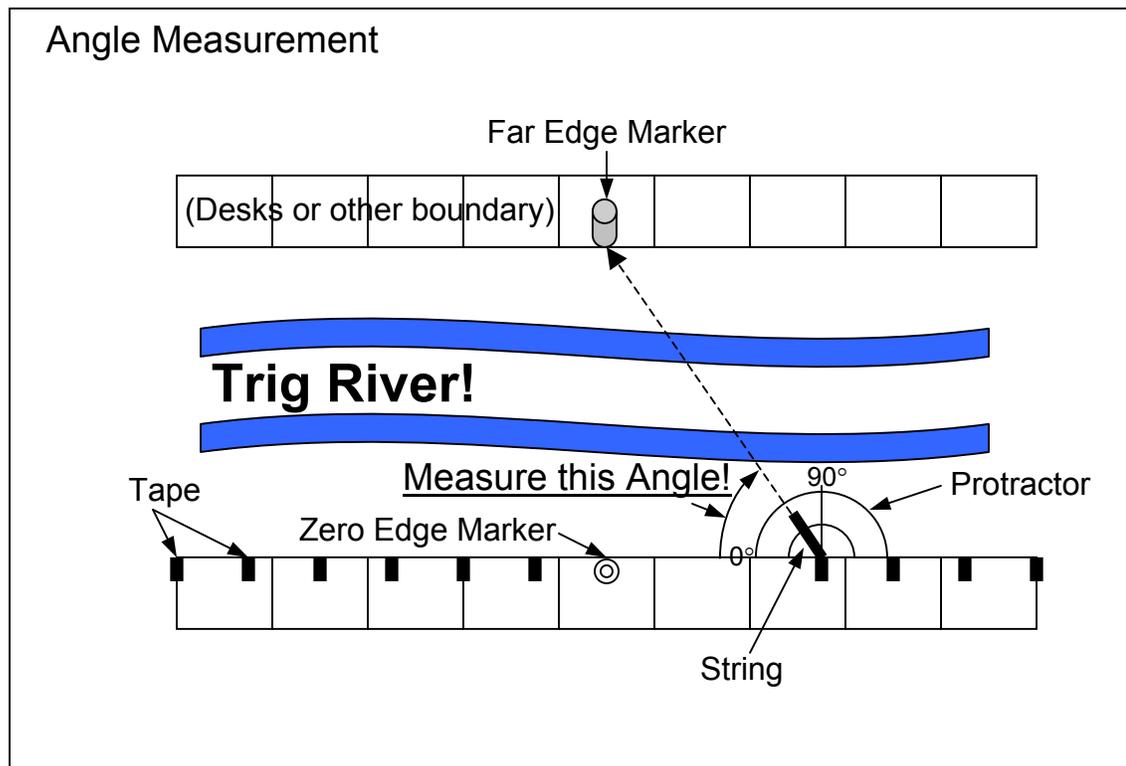


Figure 2 – Angle Measurement

8. Partners switch jobs and make a second measurement. Record on worksheet.
9. Complete the worksheet calculations. (Leave the desks and markers in place.)
10. Have students compare their estimate of the river width to the actual measurement. How close was it?
11. Question and answer #1: Were students that were closer to the zero marker more or less accurate than those further away? (Answer: Students close to the zero marker should be less accurate because the values of the tangents of angles close

to 90 degrees become large quickly and a small error in the angle measurement results in a large distance error. Note that the same problem would be seen as the measured angle approached zero degrees, but a student would have to be infinitely far away for that.)

12. Question and answer #2: Could this method be used in the wilderness if you did not have a calculator or Trig Tables? (Answer: It is not easy to memorize the tangent values for all angles but one value is very easy to remember. Have the students find $\tan(45)$ and then explain why they get a simple answer.)

Math Skills Reinforced —

6th, 7th, 8th: Students use right triangle trigonometry and angle measurements to calculate distances. Averaging, comparison of numbers, and data trends are also explored. Students will also estimate distances.

7th, 8th: Convert from US Customary to Metric units (extension activity)

8th: Creating a bar graph (extension activity)

Activity Troubleshooting Tips —

Students may get confused on the trigonometry involved in this activity. After all of the students have tried to figure out the distance across the river, choose one of the tape marks and have the class walk through the activity together for reinforcement.

Activity Desired Student Outcomes —

After this activity, students should understand how useful geometry and triangles are in determining distances. They should realize that different triangles could be used to find the same distance.

Activity Assessment & Evaluation —

Pre-Activity Assessment

- Discussion:
 - Is it possible to determine the width of a river without crossing it? (Allow discussion and entertain any creative ideas: walk through the river, throw a rope across, use stepping-stones, etc. Then state that this river does not allow those solutions because it is too deep, the current is too swift, it is too wide, you do not have that tool, etc.) See Part C, *With the Students*.
- Prediction: Have students make an estimate of how wide the river is.

Activity Embedded Assessment

- Trig Calculations: Students follow and complete the attached worksheet.

Post-Activity Assessment

- Questions:
 - Were students closer to the zero marker more or less accurate than those further away? (Answer: Students close to the zero marker should be less

accurate because the values of the tangents of angles close to 90 degrees become large quickly and a small error in the angle measurement results in a large distance error. Note that the same problem would be seen as the measured angle approached zero degrees, but a student would have to be infinitely far away for that.)

- Could this method be used in the wilderness if you didn't have a calculator or Trig Tables? (Answer: It is not easy to memorize the tangent values for all angles but one value is very easy to remember. Have the students find $\tan(45)$ and then explain why they get a simple answer.)

Suggestions to Scale Activity for Grades 6 to 8 —

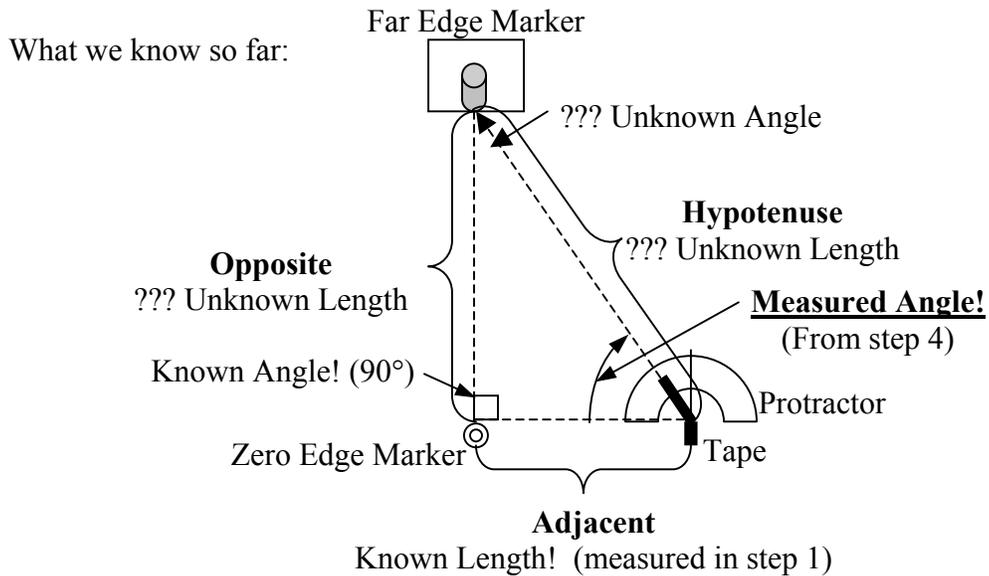
- 6th Grade: Do activity as is.
- 7th Grade: Have the students use both metric and English units to measure the distance from the zero marker. Compare the two results at the end.
- 8th Grade: Have the students determine the difference between their estimate and the actual value. Then as a class, plot all the differences (in estimated versus actual values) vs. the distances from the zero marker. Do the estimates become more or less accurate when measured near or far from the zero marker? Is there an *optimal distance* from the zero marker that minimizes the error?

Trig River Worksheet

Name: _____

Date: _____

- Your distance from the Zero Edge Marker: _____
- 1st pencil angle measured on the protractor: _____ (degrees)
- 2nd pencil angle measured on the protractor: _____ (degrees)
- Find the *average* of these two angle measurements by adding them together and dividing by 2:
 $(1^{\text{st}} \text{ Angle}) \text{ ______} + (2^{\text{nd}} \text{ Angle}) \text{ ______} = \text{ ______} \div 2 = \text{ ______} \text{ (degrees)}$



We know that the length of the side *Adjacent* to the measured angle and we do not know the length of the side *Opposite* the angle or the length of the *Hypotenuse*.

- Which of the trigonometric functions use the side we know and the side we want to know in our “river”? Circle one: **Sin = Opp** **Cos = Adj**

Tan = Opp

Hyp

Hyp

Adj

- What is the Tan of your measured angle (use a calculator or chart)? _____
- Now you have one unknown in your equation, and you can solve it!
 Length of Adjacent Side x Tan value of your angle = Length of Opposite Side!
 _____ x _____ = ?

Write your answer here: _____

This is the width of the Trig River!

Navigation Trigonometry Tables

Angle (deg)	Sine	Cosine	Tangent		Angle (deg)	Sine	Cosine	Tangent
0	0.0000	1.0000	0.0000		Continued			
1	0.0175	0.9998	0.0175		46	0.7193	0.6947	1.0355
2	0.0349	0.9994	0.0349		47	0.7314	0.6820	1.0724
3	0.0523	0.9986	0.0524		48	0.7431	0.6691	1.1106
4	0.0698	0.9976	0.0699		49	0.7547	0.6561	1.1504
5	0.0872	0.9962	0.0875		50	0.7660	0.6428	1.1918
6	0.1045	0.9945	0.1051		51	0.7771	0.6293	1.2349
7	0.1219	0.9925	0.1228		52	0.7880	0.6157	1.2799
8	0.1392	0.9903	0.1405		53	0.7986	0.6018	1.3270
9	0.1564	0.9877	0.1584		54	0.8090	0.5878	1.3764
10	0.1736	0.9848	0.1763		55	0.8192	0.5736	1.4281
11	0.1908	0.9816	0.1944		56	0.8290	0.5592	1.4826
12	0.2079	0.9781	0.2126		57	0.8387	0.5446	1.5399
13	0.2250	0.9744	0.2309		58	0.8480	0.5299	1.6003
14	0.2419	0.9703	0.2493		59	0.8572	0.5150	1.6643
15	0.2588	0.9659	0.2679		60	0.8660	0.5000	1.7321
16	0.2756	0.9613	0.2867		61	0.8746	0.4848	1.8040
17	0.2924	0.9563	0.3057		62	0.8829	0.4695	1.8807
18	0.3090	0.9511	0.3249		63	0.8910	0.4540	1.9626
19	0.3256	0.9455	0.3443		64	0.8988	0.4384	2.0503
20	0.3420	0.9397	0.3640		65	0.9063	0.4226	2.1445
21	0.3584	0.9336	0.3839		66	0.9135	0.4067	2.2460
22	0.3746	0.9272	0.4040		67	0.9205	0.3907	2.3559
23	0.3907	0.9205	0.4245		68	0.9272	0.3746	2.4751
24	0.4067	0.9135	0.4452		69	0.9336	0.3584	2.6051
25	0.4226	0.9063	0.4663		70	0.9397	0.3420	2.7475
26	0.4384	0.8988	0.4877		71	0.9455	0.3256	2.9042
27	0.4540	0.8910	0.5095		72	0.9511	0.3090	3.0777
28	0.4695	0.8829	0.5317		73	0.9563	0.2924	3.2709
29	0.4848	0.8746	0.5543		74	0.9613	0.2756	3.4874
30	0.5000	0.8660	0.5774		75	0.9659	0.2588	3.7321
31	0.5150	0.8572	0.6009		76	0.9703	0.2419	4.0108
32	0.5299	0.8480	0.6249		77	0.9744	0.2250	4.3315
33	0.5446	0.8387	0.6494		78	0.9781	0.2079	4.7046
34	0.5592	0.8290	0.6745		79	0.9816	0.1908	5.1446
35	0.5736	0.8192	0.7002		80	0.9848	0.1736	5.6713
36	0.5878	0.8090	0.7265		81	0.9877	0.1564	6.3138
37	0.6018	0.7986	0.7536		82	0.9903	0.1392	7.1154
38	0.6157	0.7880	0.7813		83	0.9925	0.1219	8.1443
39	0.6293	0.7771	0.8098		84	0.9945	0.1045	9.5144
40	0.6428	0.7660	0.8391		85	0.9962	0.0872	11.4301
41	0.6561	0.7547	0.8693		86	0.9976	0.0698	14.3007
42	0.6691	0.7431	0.9004		87	0.9986	0.0523	19.0811
43	0.6820	0.7314	0.9325		88	0.9994	0.0349	28.6363
44	0.6947	0.7193	0.9657		89	0.9998	0.0175	57.2900
45	0.7071	0.7071	1.0000		90	1.0000	0.0000	Infinity
Angle (deg)	Sine	Cosine	Tangent		Angle (deg)	Sine	Cosine	Tangent